

IIT - JEE ADVANCED - 2012

PAPER-1 [Code – 8]

PART - III: MATHEMATICS

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

41. If $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then
(A) $a = 1, b = 4$
(C) $a = 2, b = -3$

- (B) $a = 1, b = -4$
(D) $a = 2, b = 3$

Sol. (B)

$$\text{Given } \lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^2 + x + 1 - ax^2 - ax - bx - b}{(x + 1)} = 4 \Rightarrow \lim_{x \rightarrow \infty} \frac{(1 - a)x^2 + (1 - a - b)x + (1 - b)}{(x + 1)} = 4$$

$$\Rightarrow 1 - a = 0 \text{ and } 1 - a - b = 4 \Rightarrow b = -4, a = 1.$$

42. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is
(A) 2^{10}
(C) 2^{12}

- (B) 2^{11}
(D) 2^{13}

Sol. (D)

$$|Q| = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix} \Rightarrow |Q| = 2^2 \cdot 2^3 \cdot 2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2 a_{31} & 2^2 a_{32} & 2^2 a_{33} \end{vmatrix}$$

$$|Q| = 2^9 \cdot 2 \cdot 2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Rightarrow |Q| = 2^{12} |P|$$

$$|Q| = 2^{13}.$$

43. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is
(A) $20(x^2 + y^2) - 36x + 45y = 0$
(C) $36(x^2 + y^2) - 20x + 45y = 0$

- (B) $20(x^2 + y^2) + 36x - 45y = 0$
(D) $36(x^2 + y^2) + 20x - 45y = 0$

Sol. (A)

Equation of the chord bisected at $P(h, k)$

$$hx + ky = h^2 + k^2 \quad \dots(i)$$

Let any point on line be $\left(\alpha, \frac{4}{5}\alpha - 4 \right)$

Equation of the chord of contact is

$$\Rightarrow \alpha x + \left(\frac{4}{5}\alpha - 4\right)y = 9 \quad \dots(ii)$$

Comparing (i) and (ii)

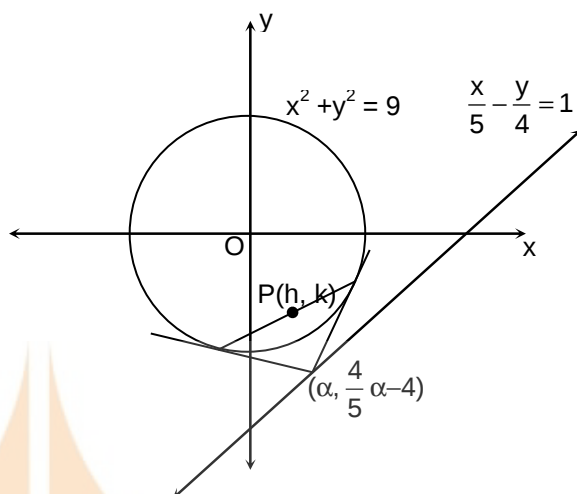
$$\frac{h}{\alpha} = \frac{k}{\frac{4}{5}\alpha - 4} = \frac{h^2 + k^2}{9}$$

$$\alpha = \frac{20h}{4h - 5k}$$

$$\text{Now, } \frac{h(4h - 5k)}{20h} = \frac{h^2 + k^2}{9}$$

$$20(h^2 + k^2) = 9(4h - 5k)$$

$$20(x^2 + y^2) - 36x + 45y = 0.$$



44. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is
 (A) 75 (B) 150
 (C) 210 (D) 243

Sol. (B)

Number of ways

$$= 3^5 - {}^3C_1 \cdot 2^5 + {}^3C_2 \cdot 1^5$$

$$= 243 - 96 + 3 = 150.$$

45. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

$$(A) -\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K \quad (B) \frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$$

$$(C) -\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K \quad (D) \frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$$

Sol. (C)

$$I = \int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$$

$$\text{Let } \sec x + \tan x = t$$

$$\Rightarrow \sec x - \tan x = 1/t$$

$$\text{Now } (\sec x \tan x + \sec^2 x) dx = dt$$

$$\sec x (\sec x + \tan x) dx = dt$$

$$\sec x dx = \frac{dt}{t}, \quad \frac{1}{2} \left(t + \frac{1}{t} \right) = \sec x$$

$$I = \frac{1}{2} \int \frac{\left(t + \frac{1}{t} \right) dt}{t^{9/2} \cdot t}$$

$$= \frac{1}{2} \int (t^{-9/2} + t^{-13/2}) dt$$

$$= \frac{1}{2} \left[\frac{t^{-9/2+1}}{-\frac{9}{2}+1} + \frac{t^{-13/2+1}}{-\frac{13}{2}+1} \right]$$



$$\begin{aligned}
&= \frac{1}{2} \left[\frac{t^{-7/2}}{-\frac{7}{2}} + \frac{t^{-11/2}}{-\frac{11}{2}} \right] \\
&= -\frac{1}{7} t^{-7/2} - \frac{1}{11} t^{-11/2} \\
&= -\frac{1}{7} \frac{1}{t^{7/2}} - \frac{1}{11} \frac{1}{t^{11/2}} \\
&= -\frac{1}{t^{11/2}} \left(\frac{1}{11} + \frac{t^2}{7} \right) = -\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + k
\end{aligned}$$

46. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, -1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is

- (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
(C) 2 (D) $2\sqrt{2}$

Sol.

(A)

D. R. of QR is 1, 4, 1

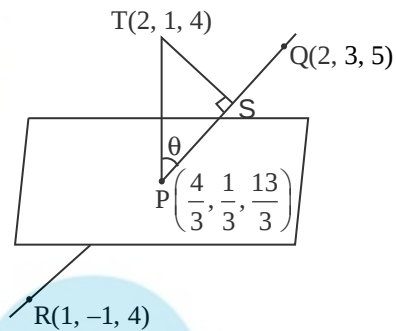
Coordinate of P $\equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3} \right)$

D. R. of PT is 2, 2, -1

Angle between QR and PT is 45°

And PT = 1

$$\Rightarrow PS = TS = \frac{1}{\sqrt{2}}$$



47. Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in \mathbb{R}$, then f is

- (A) differentiable both at $x = 0$ and at $x = 2$
(B) differentiable at $x = 0$ but not differentiable at $x = 2$
(C) not differentiable at $x = 0$ but differentiable at $x = 2$
(D) differentiable neither at $x = 0$ nor at $x = 2$

Sol.

(B)

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 \left| \cos \frac{\pi}{h} \right| - 0}{h}$$

$$= \lim_{h \rightarrow 0} h \cos \left(\frac{\pi}{h} \right) = 0$$

so, $f(x)$ is differentiable at $x = 0$

$$f'(2^+) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(2+h)^2 \left| \cos \frac{\pi}{2+h} \right| - 0}{h}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{(2+h)^2 \cos\left(\frac{\pi}{2+h}\right)}{h} \\
 f'(2^+) &= \lim_{h \rightarrow 0} \frac{(2+h)^2}{h} \sin\left(\frac{\pi}{2} - \frac{\pi}{2+h}\right) \\
 &= \lim_{h \rightarrow 0} \frac{(2+h)^2}{h} \sin\left[\frac{\pi \cdot h}{2(2+h)}\right] \\
 &= \lim_{h \rightarrow 0} \frac{(2+h)^2}{\pi h} \sin \frac{\pi h}{2(2+h)} \times \frac{\pi}{2(2+h)} = \pi
 \end{aligned}$$

$$\begin{aligned}
 \text{Again, } f'(2^-) &= \lim_{h \rightarrow 0} \frac{f(2-h) - f(2)}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{(2-h)^2 \cos\left(\frac{\pi}{2-h}\right)}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{-(2-h)^2 \cos\left(\frac{\pi}{2-h}\right)}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{(2-h)^2 \sin\left[\frac{\pi}{2} - \frac{\pi}{2-h}\right]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(2-h)^2}{h} \cdot \sin\left[\frac{-\pi h}{2(2-h)}\right] \\
 &= -\lim_{h \rightarrow 0} \frac{(2-h)^2}{\pi h} \cdot \sin \frac{\pi h}{2(2-h)} \times \frac{\pi}{2(2-h)} = -\pi
 \end{aligned}$$

48. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value

- (A) -1 (B) $\frac{1}{3}$
 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Sol.

(D)

Given equation is $z^2 + z + 1 - a = 0$

Clearly this equation do not have real roots if

$D < 0$

$\Rightarrow 1 - 4(1 - a) < 0$

$\Rightarrow 4a < 3$

$a < \frac{3}{4}$.

49. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes.

Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R . The eccentricity of the ellipse E_2 is

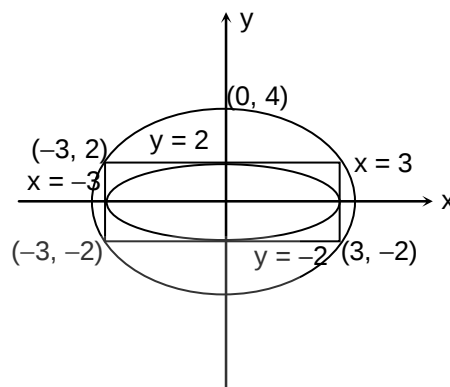
- (A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

Sol. (C)Equation of ellipse is $(y + 2)(y - 2) + \lambda(x + 3)(x - 3) = 0$ It passes through $(0, 4) \Rightarrow \lambda = \frac{4}{3}$ Equation of ellipse is $\frac{x^2}{12} + \frac{y^2}{16} = 1$

$$e = \frac{1}{2}.$$

**Alternate**Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as it is passing through $(0, 4)$ and $(3, 2)$.

So, $b^2 = 16$ and $\frac{9}{a^2} + \frac{4}{16} = 1$

$$\Rightarrow a^2 = 12$$

So, $12 = 16(1 - e^2)$

$$\Rightarrow e = 1/2.$$

50. The function $f : [0, 3] \rightarrow [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is
 (A) one-one and onto (B) onto but not one-one
 (C) one-one but not onto (D) neither one-one nor onto

Sol. (B)

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$f'(x) = 6x^2 - 30x + 36$$

$$= 6(x^2 - 5x + 6)$$

$$= 6(x - 2)(x - 3)$$

 $f(x)$ is increasing in $[0, 2]$ and decreasing in $[2, 3]$ $f(x)$ is many one

$$f(0) = 1$$

$$f(2) = 29$$

$$f(3) = 28$$

Range is $[1, 29]$ Hence, $f(x)$ is many-one-onto**SECTION II : Multiple Correct Answer(s) Type**

This section contains 5 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

51. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are
 (A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$
 (C) $(3\sqrt{3}, -2\sqrt{2})$ (D) $(-3\sqrt{3}, 2\sqrt{2})$

Sol. (A, B)

Slope of tangent = 2

The tangents are $y = 2x \pm \sqrt{9 \times 4 - 4}$

i.e., $2x - y = \pm 4\sqrt{2}$

$$\Rightarrow \frac{x}{2\sqrt{2}} - \frac{y}{4\sqrt{2}} = 1 \text{ and } -\frac{x}{2\sqrt{2}} + \frac{y}{4\sqrt{2}} = 1$$

Comparing it with $\frac{xx_1}{9} - \frac{yy_1}{4} = 1$

We get point of contact as $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Alternate:

Equation of tangent at P (θ) is $\left(\frac{\sec \theta}{3}\right)x - \left(\frac{\tan \theta}{2}\right)y = 1$

$$\Rightarrow \text{Slope} = \frac{2 \sec \theta}{3 \tan \theta} = 2$$

$$\Rightarrow \sin \theta = \frac{1}{3}$$

$$\Rightarrow \text{points are } \left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \text{ and } \left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right).$$

52. Let $\theta, \phi \in [0, 2\pi]$ be such that $2 \cos \theta (1 - \sin \phi) = \sin^2 \theta \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) \cos \phi - 1$, $\tan(2\pi - \theta) > 0$ and

$-1 < \sin \theta < -\frac{\sqrt{3}}{2}$. Then ϕ **cannot** satisfy

(A) $0 < \phi < \frac{\pi}{2}$

(B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$

(C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$

(D) $\frac{3\pi}{2} < \phi < 2\pi$

Sol. (A, C, D)

$$2 \cos \theta (1 - \sin \phi) = \frac{2 \sin^2 \theta}{\sin \theta} \cos \phi - 1 = 2 \sin \theta \cos \phi - 1$$

$$2 \cos \theta - 2 \cos \theta \sin \phi = 2 \sin \theta \cos \phi - 1$$

$$2 \cos \theta + 1 = 2 \sin(\theta + \phi)$$

$$\tan(2\pi - \theta) > 0 \Rightarrow \tan \theta < 0 \text{ and } -1 < \sin \theta < -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$$

$$\frac{1}{2} < \sin(\theta + \phi) < 1$$

$$\Rightarrow 2\pi + \frac{\pi}{6} < \theta + \phi < \frac{5\pi}{6} + 2\pi$$

$$2\pi + \frac{\pi}{6} - \theta_{\max} < \phi < 2\pi + \frac{5\pi}{6} - \theta_{\min}$$

$$\frac{\pi}{2} < \phi < \frac{4\pi}{3}.$$

53. If $y(x)$ satisfies the differential equation $y' - y \tan x = 2x \sec x$ and $y(0) = 0$, then

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$

(B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

$$(C) \ y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$$

$$(D) \ y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$$

Sol. (A, D)

$$\frac{dy}{dx} - y \tan x = 2x \sec x$$

$$\cos x \frac{dy}{dx} + (-\sin x)y = 2x$$

$$\frac{d}{dx}(y \cos x) = 2x$$

$$y(x) \cos x = x^2 + c, \text{ where } c = 0 \text{ since } y(0) = 0$$

$$\text{when } x = \frac{\pi}{4}, y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}, \text{ when } x = \frac{\pi}{3}, y\left(\frac{\pi}{3}\right) = \frac{2\pi^2}{9}$$

$$\text{when } x = \frac{\pi}{4}, y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}} + \frac{\pi}{\sqrt{2}}$$

$$\text{when } x = \frac{\pi}{3}, y'\left(\frac{\pi}{3}\right) = \frac{2\pi^2}{3\sqrt{3}} + \frac{4\pi}{3}$$

54. A ship is fitted with three engines E_1, E_2 and E_3 . The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is(are) true?

$$(A) \ P[X_1^c | X] = \frac{3}{16}$$

$$(B) \ P[\text{Exactly two engines of the ship are functioning} | X] = \frac{7}{8}$$

$$(C) \ P[X | X_2] = \frac{5}{16}$$

$$(D) \ P[X | X_1] = \frac{7}{16}$$

Sol. (B, D)

$$P(X_1) = \frac{1}{2}, P(X_2) = \frac{1}{4}, P(X_3) = \frac{1}{4}$$

$$P(X) = P(X_1 \cap X_2 \cap X_3^c) + P(X_1 \cap X_2^c \cap X_3) + P(X_1^c \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap X_3) = \frac{1}{4}$$

$$(A) \ P(X_1^c | X) = \frac{P(X \cap X_1^c)}{P(X)} = \frac{\frac{1}{32}}{\frac{1}{4}} = \frac{1}{8}$$

$$(B) \ P[\text{exactly two engines of the ship are functioning} | X] = \frac{\frac{7}{32}}{\frac{1}{4}} = \frac{7}{8}$$

$$(C) \ P\left(\frac{X}{X_2}\right) = \frac{\frac{5}{32}}{\frac{1}{4}} = \frac{5}{8}$$

$$(D) \ P\left(\frac{X}{X_1}\right) = \frac{\frac{7}{32}}{\frac{1}{2}} = \frac{7}{16}$$



55. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$. Then

(A) $S \geq \frac{1}{e}$

(B) $S \geq 1 - \frac{1}{e}$

(C) $S \leq \frac{1}{4} \left(1 + \frac{1}{\sqrt{e}} \right)$

(D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}} \left(1 - \frac{1}{\sqrt{2}} \right)$

Sol. (A, B, D)

$S > \frac{1}{e}$ (As area of rectangle OCDS = $1/e$)

Since $e^{-x^2} \geq e^{-x} \forall x \in [0, 1]$

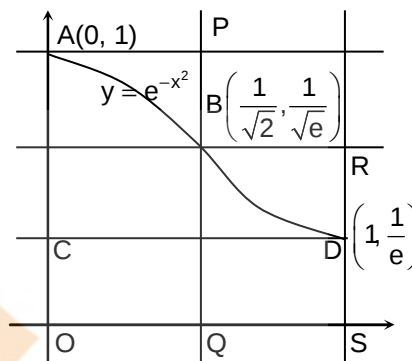
$\Rightarrow S > \int_0^1 e^{-x} dx = \left(1 - \frac{1}{e} \right)$

Area of rectangle OAPQ + Area of rectangle QBRS > S

$S < \frac{1}{\sqrt{2}} \left(1 + \left(1 - \frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{e}} \right) \right)$

Since $\frac{1}{4} \left(1 + \frac{1}{\sqrt{e}} \right) < 1 - \frac{1}{e}$

Hence, (C) is incorrect.



SECTION III : Integer Answer Type

This section contains 5 questions. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive).

56. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

Sol. (3)

As, $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 3(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2) - |\vec{a} + \vec{b} + \vec{c}|^2$

$\Rightarrow 3 \times 3 - |\vec{a} + \vec{b} + \vec{c}|^2 = 9$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 0 \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{b} + \vec{c} = -\vec{a}$

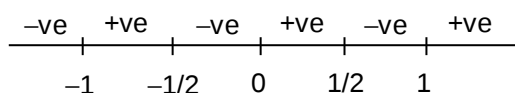
$\Rightarrow |2\vec{a} + 5(\vec{b} + \vec{c})| = |-3\vec{a}| = 3|\vec{a}| = 3$.

57. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Sol. (5)

$f'(x) = \frac{|x|}{x} + \frac{|x^2 - 1|}{x^2 - 1} \cdot (2x)$

$= \begin{cases} 2x-1 & , \quad x < -1 \\ -(2x+1) & , \quad -1 < x < 0 \\ 1-2x & , \quad 0 < x < 1 \\ 2x+1 & , \quad x > 1 \end{cases}$



So, $f'(x)$ changes sign at points

$$x = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$$

so, total number of points of local maximum or minimum is 5.

58. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Sol. (4)

The parabola is $x = 2t^2, y = 4t$

Solving it with the circle we get :

$$4t^4 + 16t^2 - 4t^2 - 16t = 0$$

$$\Rightarrow t^4 + 3t^2 - 4t = 0 \Rightarrow t = 0, 1$$

so, the points P and Q are (0, 0) and (2, 4) which are also diametrically opposite points on the circle. The focus is $S \equiv (2, 0)$.

$$\text{The area of } \Delta PQS = \frac{1}{2} \times 2 \times 4 = 4.$$

59. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is

Sol. (9)

$$\text{Let } p'(x) = k(x-1)(x-3)$$

$$\Rightarrow p(x) = k \left(\frac{x^3}{3} - 2x^2 + 3x \right) + c$$

$$\text{Now, } p(1) = 6 \Rightarrow \frac{4}{3}k + c = 6$$

$$\text{also, } p(3) = 2 \Rightarrow c = 2$$

$$\text{so, } k = 3, \text{ so, } p'(0) = 3k = 9.$$

60. The value of $6 + \log_{3/2} \left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \dots \right)$ is

Sol. (4)

$$\text{Let } \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \dots = y$$

$$\text{So, } 4 - \frac{1}{3\sqrt{2}} y = y^2 \quad (y > 0)$$

$$\Rightarrow y^2 + \frac{1}{3\sqrt{2}} y - 4 = 0 \Rightarrow y = \frac{8}{3\sqrt{2}}$$

$$\text{so, the required value is } 6 + \log_{3/2} \left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}} \right)$$

$$= 6 + \log_{3/2} \frac{4}{9} = 6 - 2 = 4.$$